Trinity College

WA Exams Practice Paper A, 2015

Question/Answer Booklet

MATHEMATICS SPECIALIST UNITS 1 AND 2

Section One: Calculator-free

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time for section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

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Section One: Calculator-free **(52 Marks)**

This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(7 marks)**

Four matrices are given by $A=\begin{bmatrix}2 & 6\end{bmatrix}$, $B=\begin{bmatrix}6 & 2\\ 2 & 4\\ 0 & -2\end{bmatrix}$, $C=\begin{bmatrix}4 & 3\\ 2 & 1\end{bmatrix}$, $D=\begin{bmatrix}-1 & 2\\ 5 & 4\end{bmatrix}$.

If possible, use these matrices to find P, Q and R below. If not possible, explain why.

(a) $P=\frac{1}{2}BD$. (2 marks)

$$P=\frac{1}{2}\begin{bmatrix}6 & 2\\ 2 & 4\\ 0 & -2\end{bmatrix}\begin{bmatrix}-1 & 2\\ 5 & 4\end{bmatrix}=\begin{bmatrix}3 & 1\\ 1 & 2\\ 0 & -1\end{bmatrix}\begin{bmatrix}-1 & 2\\ 5 & 4\end{bmatrix}=\begin{bmatrix}2 & 10\\ 9 & 10\\ -5 & -4\end{bmatrix}$$

(b) $Q=2C-3D$. (2 marks)

$$Q=2\begin{bmatrix}4 & 3\\ 2 & 1\end{bmatrix}-3\begin{bmatrix}-1 & 2\\ 5 & 4\end{bmatrix}=\begin{bmatrix}11 & 0\\ -11 & -10\end{bmatrix}$$

(c) R, where $RC=A$. (3 marks)

$$RC=A \Rightarrow R=AC^{-1}$$

$$R=\begin{bmatrix}2 & 6\end{bmatrix}\begin{bmatrix}4 & 3\\ 2 & 1\end{bmatrix}^{-1}$$

$$=\begin{bmatrix}2 & 6\end{bmatrix}\times\frac{1}{-2}\times\begin{bmatrix}1 & -3\\ -2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}-1 & -3\end{bmatrix}\times\begin{bmatrix}1 & -3\\ -2 & 4\end{bmatrix}$$

$$=\begin{bmatrix}5 & -9\end{bmatrix}$$

See next page

Question 2 **(7 marks)**

(a) Use the identity $\cos(\theta \pm \varphi) = \cos\theta\cos\varphi \mp \sin\theta\sin\varphi$ to prove that $\cos 2\theta = 2\cos^2\theta - 1$. (3 marks)

$$\cos(\theta + \theta) = \cos\theta\cos\theta - \sin\theta\sin\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= \cos^2\theta - (1 - \cos^2\theta)$$
$$= 2\cos^2\theta - 1$$

(b) Solve $\cos\theta = \cos 2\theta$ over the domain $0 \le \theta \le 2\pi$. (4 marks)

$$\cos\theta = \cos 2\theta$$
$$\cos\theta = 2\cos^2\theta - 1$$
$$0 = 2\cos^2\theta - \cos\theta - 1$$
$$0 = (2\cos\theta + 1)(\cos\theta - 1)$$

$2\cos\theta + 1 = 0$ $\qquad$ $\cos\theta - 1 = 0$

$\cos\theta = -\frac{1}{2}$ $\qquad$ $\cos\theta = 1$

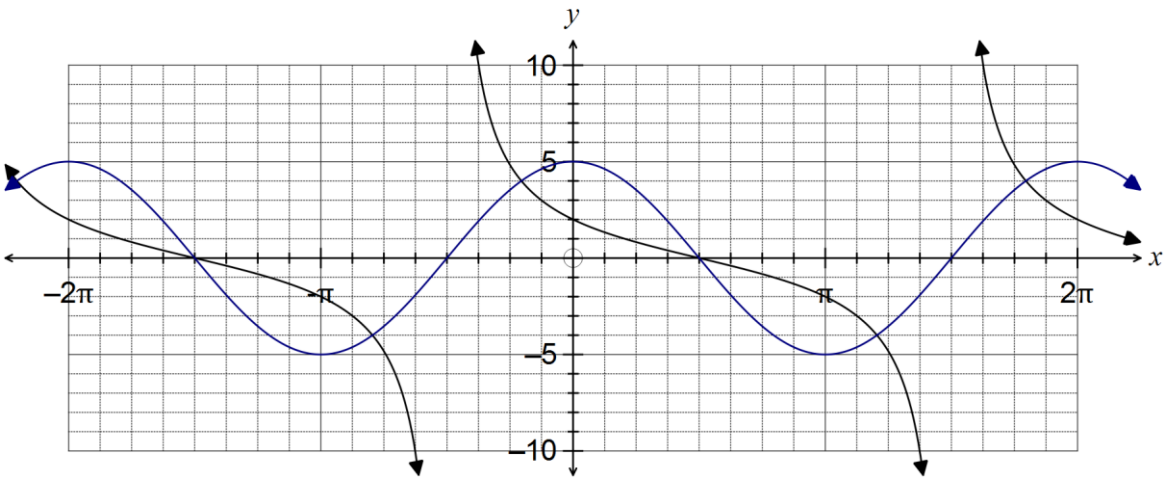
$\theta = \frac{2\pi}{3}, \ \frac{4\pi}{3}$ $\qquad$ $\theta = 0, \ 2\pi$

$$\theta = 0, \ \frac{2\pi}{3}, \ \frac{4\pi}{3}, \ 2\pi$$

See next page

Question 3 **(7 marks)**

The function $f(x) = a \cdot \tan(b(x + c))$ has been graphed below.



(a) Determine the values of the constants a, b and c. (4 marks)

Period of $\tan x$ is π, $f(x)$ is 2π, so $b = \frac{1}{2}$.

$y = \tan x$ has a root at $x = 0$. $f(x)$ has root at $\frac{\pi}{2}$, so $c = -\frac{\pi}{2}$.

$f(0) = a\tan(\frac{1}{2}(0 - \frac{\pi}{2}))$

$= a\tan(-\frac{\pi}{4})$ but $f(0) = 2$ so $a = -2$

$= -a$

$a = -2,\ b = \frac{1}{2},\ c = -\frac{\pi}{2}$

(b) On the same axes, sketch the graph of $y = 5\cos x$. (2 marks)

(c) State the number of solutions to the equation $5\cos x = f(x)$ over the domain $0 \le x \le 2\pi$. (1 mark)

3 solutions

Question 4 **(8 marks)**

Two vectors are given by $\mathbf{p} = 6\mathbf{i} + (3x + 7)\mathbf{j}$ and $\mathbf{q} = (x + 2)\mathbf{i} + 4\mathbf{j}$.

(a) If $x = -1$, find a vector parallel to $\mathbf{q}$ that has the same magnitude as $\mathbf{p}$. Give your answer in exact form. (4 marks)

$\mathbf{p} = 6\mathbf{i} + 4\mathbf{j}$ $\quad$ $\mathbf{q} = \mathbf{i} + 4\mathbf{j}$

$|\mathbf{p}| = 2\sqrt{13}$ $\quad$ $|\mathbf{q}| = \sqrt{17}$

Required vector is $= \dfrac{2\sqrt{13}}{\sqrt{17}}(\mathbf{i} + 4\mathbf{j})$

$= \dfrac{2\sqrt{221}}{17}(\mathbf{i} + 4\mathbf{j})$

(or in opposite direction)

(b) Determine all possible values of x so that $\mathbf{p}$ and $\mathbf{q}$ are parallel. (4 marks)

$$\frac{6}{x+2} = \frac{3x+7}{4}$$

$$24 = (3x + 7)(x + 2)$$

$$3x^2 + 13x - 10 = 0$$

$$(3x - 2)(x + 5) = 0$$

$$x = -5,\ x = \tfrac{2}{3}$$

Question 5 **(8 marks)**

(a) If $z = 2 - i$, determine each of the following in the form $x + yi$.

(i) $z(1 + \overline{z})$. (2 marks)

$$\begin{aligned} z(1+\overline{z}) &= (2-i)(1+2+i) \\ &= (2-i)(3+i) \\ &= 6 + 2i - 3i - i^2 \\ &= 7 - i \end{aligned}$$

(ii) $\dfrac{z}{2-z}$. (2 marks)

$$\begin{aligned} \frac{z}{2-z} &= \frac{2-i}{2-(2-i)} \\ &= \frac{2-i}{i} \times \frac{-i}{-i} \\ &= -1 - 2i \end{aligned}$$

(b) Determine both complex solutions to the equation $x^2 + 2x + 5 = 0$. (2 marks)

$$\begin{aligned} (x+1)^2 &= -4 \\ &= -1 \pm \sqrt{-4} \\ &= -1 + 2i, \ \ -1 - 2i \end{aligned}$$

(c) Determine the complex number w if $w + iw = 1 + 7i$. (2 marks)

Let $w = a + bi$

$$a + bi + i(a + bi) = 1 + 7i$$

$$a - b + (a + b)i = 1 + 7i$$

$$a - b = 1$$

$$a + b = 7$$

$$2a = 8 \ \Rightarrow \ a = 4, \ b = 3$$

$$w = 4 + 3i$$

See next page

Question 6 **(8 marks)**

(a) Determine the least number of people required in a group to be certain that at least ten of them share the same birth month. (2 marks)

$(10-1)\times 12=108$

$108+1=109$

(b) A shop has a stock of 188 bottles of wine from six different producers. Prove that the stock includes at least 32 bottles from one of the producers. (2 marks)

Fewest bottles from one producer when stock shared equally between them: $188\div 6=31r2$ - so 31 bottles per producer with 2 left over. So at least one producer must have 31 + 1 = 32 bottles in stock.

(c) Determine how many positive even integers smaller than 3000 can be made using some or all of the digits 1, 2, 3, 4 and 5, without repetition. (4 marks)

4 digit starting with 1: $1\times 2\times 3\times 2=12$
4 digit starting with 2: $1\times 1\times 3\times 2=6$
3 digit start with even: $2\times 1\times 3=6$
3 digit start with odd: $3\times 2\times 3=18$
2 digit start with even: $2\times 1=2$
2 digit start with odd: $3\times 2=6$
1 digit: 2

Total: $12+6+6+18+2+6+2=52$

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Question 7 **(7 marks)**

The three points P, Q and T have position vectors $2\mathbf{i} - 3\mathbf{j}$, $6\mathbf{i} + 9\mathbf{j}$ and $x\mathbf{i} + 2\mathbf{j}$ respectively.

(a) Show that the vector $\mathbf{i} + 3\mathbf{j}$ is parallel to $\overrightarrow{PQ}$ and state the exact magnitude $\mathbf{i} + 3\mathbf{j}$.

(2 marks)

$$\overrightarrow{PQ} = \begin{bmatrix} 6 \\ 9 \end{bmatrix} - \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ 12 \end{bmatrix}$$

$$= 4\begin{bmatrix} 1 \\ 3 \end{bmatrix} \Rightarrow \text{parallel}$$

$$\sqrt{1^2 + 3^2} = \sqrt{10}$$

(b) Determine the value of x if the angle between $\overrightarrow{OT}$ and $\overrightarrow{PQ}$ is $45°$, where O is the origin.

(5 marks)

Direction of PQ is (1,3)

$$\cos 45° = \frac{(x, 2) \bullet (1, 3)}{|(x, 2)| \times |(1, 3)|}$$

$$\frac{1}{\sqrt{2}} = \frac{x + 6}{\sqrt{x^2 + 4}\sqrt{10}}$$

$$10(x^2 + 4) = 2(x + 6)^2$$

$$5x^2 + 20 = (x + 6)^2$$

$$4x^2 - 12x - 16 = 0$$

$$(x - 4)(x + 1) = 0$$

$$x = 4 \text{ or } x = -1$$

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____